

Detection of Earth-like planets around nearby stars using a petal-shaped occulter

Supplementary Information

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The main result we are publishing is an improved formulation for diffraction control in certain circumstances. In the paper we introduced a new formulation and showed how the improvement was sufficiently large to, for the first time, make occulters powerful enough to detect Earth-like planets with designs that can be implemented using today's technology.

Our improvement in capability is based on the well established Fresnel equations that were demonstrated in detail over 150 years ago. Thus the suppression of diffraction can be demonstrated theoretically by reliable physics and mathematics.

Because we chose to concentrate on the implications of the apodization for planet finding in the main paper, we have placed the mathematics in this supplemental document. We show the mathematics lead to the conclusion of high contrast and we show the tolerances required are achievable.

APODIZING MASKS

The search for the solution to the high contrast occulter must be carried out with the full complexity of the Fresnel regime. A Fraunhofer solution implies that, to good

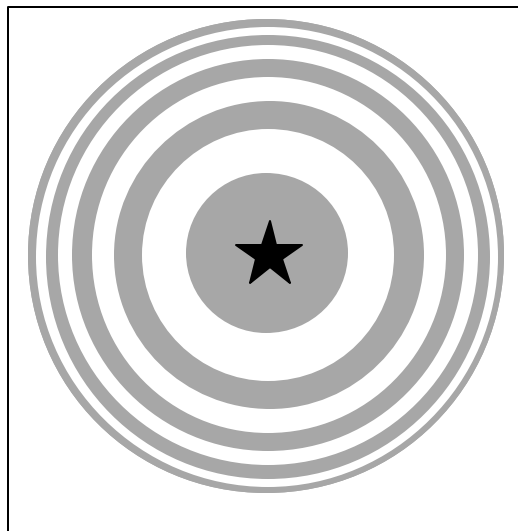


Figure 1: The central Fresnel zone and the eight inner half zones are shown schematically. The dark star in the centre represents a mask that is confined to the region where the Fraunhofer approximation can be used. It is clear that such a mask will integrate out to a net positive contribution in the focal plane.

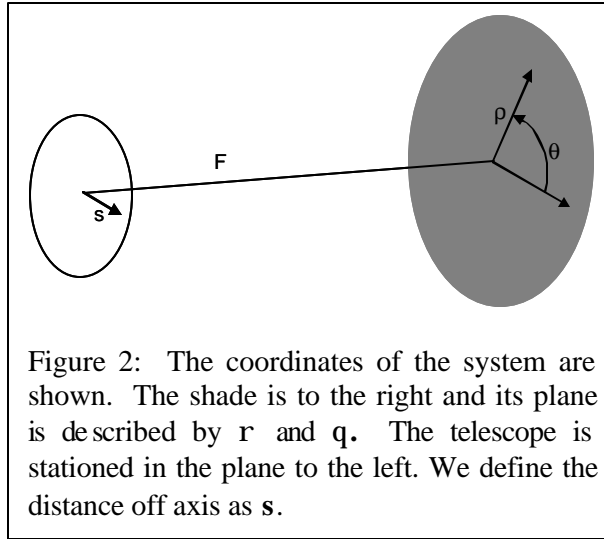
approximation, all the rays impinging upon the mask reach the axial position in the focal plane with the same phase. If one plots the Fresnel rings of the surface, as in Figure 1, one can see a Fraunhofer solution would require the mask to be restricted to the central zone. Consequently, there will always be some net positive contribution in the focal plane. To achieve a net zero electric field in the focal plane, the integral must extend out of the central zone at least into the first negative half zone.

The mathematical formulation of the Huygens-Fresnel principle can be found in most general optics textbooks (e.g. Hecht¹) The law states that the electric field at some focal plane, a distance r from a plane aperture, illuminated by a uniform plane wave from infinity is given by:

$$E = \frac{E_0}{i\lambda r} \iint A e^{ikr} dS \quad (1)$$

where the integration is over the surface S . E_0 is the strength of the electric field of the radiation incident from infinity onto the surface and r is the distance from each point on the surface to the point in the focal plane that is being evaluated. k is the usual $2\pi/\lambda$ and A is the apodization function on the occulter plane.

Figure 2 defines our coordinate system. F is the distance from mask



to focal plane. ρ is the radius on the mask, and θ its angle. s is the distance off axis on the focal plane. Then, following the usual Fresnel approximation for large F

$$E = \frac{E_0 e^{ikF}}{i\lambda F} e^{\frac{iks^2}{2F}} \int_0^\infty e^{\frac{ikr^2}{2F}} r \int_0^{2\pi} A(\mathbf{q}, \mathbf{r}) e^{\frac{ikr s \cos q}{F}} dq d\mathbf{r} \quad (2)$$

It has been shown that in many cases of deep shadow apodization, a circularly symmetric apodization may be well approximated by a binary function that appears like petals of a flower². We proceed under this assumption and later verify its appropriateness. In the case of a circularly symmetric apodization we can first integrate over angle, finding

$$E = \frac{E_0 k e^{ikF} e^{\frac{iks^2}{2F}}}{iF} \int_0^\infty e^{\frac{ikr^2}{2F}} A(\mathbf{r}) J_0\left(\frac{k\mathbf{r}s}{F}\right) r d\mathbf{r} \quad (3)$$

If $A(\rho)$ is unity to some radius a , and zero beyond, and $ik\rho^2/2F$ is small, then this integral leads to the familiar Airy disk that describes the point spread function of the typical diffraction-limited telescope.

To evaluate the diffraction properties of a mask we must use the Fresnel integral, integrating over the entire area that is open to the sky. Since we are designing a mask that covers only a tiny solid angle around the direction to the star, we would need to perform an integral that sums the wavefronts over the entire sky. This is clearly impractical, so we employ Babinet's principle¹ to ease the mathematics.

Babinet's principle states that:

$$E_0 = E_1 + E_2 \quad (4)$$

where E_0 is the electric field of the signal in the focal plane, unimpeded by a mask, E_1 is the field at the focal plane obtained by integrating the Fresnel equations over the shape of the mask as if it were an aperture, and E_2 is the field obtained by integrating over all directions outside the mask.

If we assume our unimpeded wave has amplitude of one and phase of zero at the mask, then

$$E_2 = e^{ikF} - E_1 \quad (5)$$

We simply seek a solution to the Fresnel integral over the shape of the mask such that

$$E_1 = e^{ikF} \quad (6)$$

We seek a solution in which the electric field integrated over the aperture yields, over some region on the focal plane, the same strength it would have had without an aperture.

For mathematical simplicity we confine ourselves to analysis of the on-axis ($s=0$) position. The width of the shadow is later calculated numerically from equation (3) as in Figure 2 of the main paper. When s is much smaller than $F/k\mathbf{r}$ across the mask, the Bessel function term remains close to one and equation (3) simplifies to

$$E = \frac{k}{iF} e^{ikF} \int_0^\infty A(\mathbf{r}) e^{\frac{ik\mathbf{r}^2}{2F}} \mathbf{r} d\mathbf{r} \quad (7)$$

So we seek a solution such that

$$\frac{k}{iF} e^{ikF} \int_0^\infty A(\mathbf{r}) e^{\frac{ik\mathbf{r}^2}{2F}} \mathbf{r} d\mathbf{r} = e^{ikF} \quad (8)$$

or

$$\frac{k}{iF} \int_0^\infty A(\mathbf{r}) e^{\frac{ik\mathbf{r}^2}{2F}} \mathbf{r} d\mathbf{r} = 1 \quad (9)$$

Equation (9) shows the difficulty of the problem. The exponential term in the integral is a strong function of wavelength. The obvious solution to the problem is to make $A(\rho)$ equal to the inverse function out to some radius a :

$$A(\mathbf{r}) = ie^{-\frac{ik\mathbf{r}^2}{2F}} \quad (10)$$

so that we need only solve

$$\frac{k}{F} \int_0^a r dr = 1 \quad (11)$$

While that solution may be simple, it is impractical. Equation (10) describes a perfectly transmitting sheet that phase delays light as a function of radius. Unfortunately, even the slightest change in incident wavelength can create a complete collapse of the nulling. A binary apodization approximation to this device is the Fresnel zone plate.

As is explained in the main paper, we have found a function that satisfies the requirements to high precision. The function is of the form:

$$A(\mathbf{r}) = e^{-\left(\frac{r-a}{b}\right)^n} \quad \text{for } \rho > a \quad (12)$$

$$A(\mathbf{r}) = 1 \quad \text{for } \rho < a$$

To investigate this effect we have once again used the Fresnel integral as in equation (9)

$$E = \frac{k}{iF} \int_0^a e^{\frac{ikr^2}{2F}} r dr + \frac{k}{iF} \int_a^\infty e^{-\frac{(r-a)^n}{b^n} + \frac{ikr^2}{2F}} r dr \quad (13)$$

To show this, we first perform a change of variable so that

$$\mathbf{a} = a \sqrt{\frac{k}{F}} \quad (14)$$

$$\mathbf{b} = b \sqrt{\frac{k}{F}} \quad (15)$$

and

$$t = r\sqrt{\frac{k}{F}} \quad (16)$$

So we find

$$E = \frac{1}{i} \int_0^a e^{\frac{it^2}{2}} t dt + \frac{1}{i} \int_a^\infty e^{\frac{it^2}{2} - \left(\frac{t-a}{b}\right)^n} t dt \quad (17)$$

And

$$E = 1 - e^{\frac{ia^2}{2}} + \frac{1}{i} \int_a^\infty e^{\frac{it^2}{2} - \left(\frac{t-a}{b}\right)^n} t dt \quad (18)$$

The next step is a change of variable to

$$x = \frac{t-a}{b} \quad (19)$$

so that

$$E = 1 - e^{\frac{ia^2}{2}} - i\mathbf{b} \int_0^\infty e^{-x^n} e^{\frac{i(\mathbf{b}x+\mathbf{a})^2}{2}} (\mathbf{b}x + \mathbf{a}) dx \quad (20)$$

Integration by parts then gives us

$$E = 1 - e^{\frac{ia^2}{2}} - e^{-x^n} e^{\frac{i(\mathbf{b}x+\mathbf{a})^2}{2}} \Big|_0^\infty - n \int_0^\infty e^{\frac{i(\mathbf{b}x+\mathbf{a})^2}{2}} e^{-x^n} x^{n-1} dx \quad (21)$$

or

$$E = 1 - n \int_0^\infty e^{\frac{i(\mathbf{b}x+\mathbf{a})^2}{2}} e^{-x^n} x^{n-1} dx \quad (22)$$

Therefore, equation (9) is satisfied except for a remainder term R. Returning to the coordinates of equation (17) we have

$$R = n \int_a^{\infty} e^{\frac{it^2}{2}} e^{-\left(\frac{t-a}{b}\right)^n} \left(\frac{t-a}{b}\right)^{n-1} dt \quad (23)$$

To evaluate this integral we once again integrate by parts:

$$R = e^{\frac{it^2}{2}} e^{-\left(\frac{t-a}{b}\right)^n} \left(\frac{t-a}{b}\right)^{n-1} \left(\frac{1}{it}\right) \Big|_a^{\infty} + \int_a^{\infty} e^{\frac{it^2}{2}} e^{-\left(\frac{t-a}{b}\right)^n} f(t) dt \quad (24)$$

where

$$f(t) = n \left(\frac{t-a}{b}\right)^{2n-2} \left(\frac{1}{it}\right) - \frac{n}{it^2} \left(\frac{t-a}{b}\right)^{n-1} + \frac{n(n-1)}{it} \left(\frac{t-a}{b}\right)^{n-2} \quad (25)$$

The first term of (24) is identically zero when evaluated from α to ∞ , as will be any term that contains both the exponential and a term of positive power in $(\tau-\alpha)/\beta$. Equation (25) has three terms, each of which must be integrated in the second term of equation (24). The first term of equation (25) has a higher power in $(\tau-\alpha)/\beta$ and as such will be a smaller term than the rest of R. The second term is similarly related to R itself, but is smaller by a factor of n/τ^2 . Thus, if $\beta^2 \gg n$ the third term will dominate. If β^2 is not larger than n then the transmission rises so quickly near $\rho=\alpha+\beta$ that the shade will start to resemble a disk, and Poisson's Spot will re-emerge.

We proceed to integrate by parts and taking the dominant term until we finally reach a term that does not evaluate to zero, and we find

$$R = \frac{n!}{b^n} \int_a^{\infty} e^{\frac{it^2}{2}} e^{-\left(\frac{t-a}{b}\right)^n} t^{1-n} dt \quad (26)$$

To approximate the value we consider that cosine terms vary rapidly and will integrate to a net of zero at some point in the first half cycle. That cycle will have a length of no more than $1/\alpha$. During this half cycle the second exponential term remains near one and the term in powers of τ will never exceed $\alpha(1-n)$. So we can expect that

$$R \leq \frac{n!}{b^n} \frac{1}{a} \left(\frac{1}{a} \right)^{n-1} = \frac{n!}{a^n b^n} \quad (27)$$

which tells us the accuracy to which the electric field can be suppressed. The square of R is approximately the contrast ratio to be expected in the deep shadow. There were many approximations made to achieve this result and they are only valid in certain parts of parameter space. A large number of small terms were dropped in the repeated integration by parts, which raises a concern as to the accuracy of equation (27). The validity of this formulation has been checked computationally and found to be reasonable when $\beta^2 \gg n$.

Again, we see that the optimally sized occulter will have α approximately equal to β . Also, to achieve high contrast, α^n must be quite large. This is clearly easier to achieve as n increases, explaining why the higher order curves give more compact solutions, just a few half zones wide. If n gets too high, there are diminishing returns as $n!$ rises and β approaches unity. Powers as high as $n=10$ or 12 can be practical.

This analysis has been carried out for the on-axis position in the focal plane,

but the telescope will have a significant size and the shadow must remain sufficiently deep at the mirror's edge. We have not yet developed a mathematical formulation for optimizing the starshade radius subject to a finite mirror size. However, given the power-law nature of the shadow depth, designing a shade by replacing a and b with $(a-$

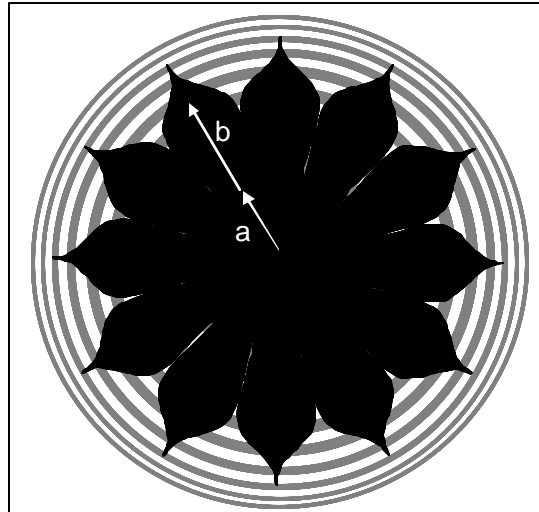


Figure 3: A twelve petal version of the starshade is shown with Fresnel zones in the background.

$R/2$) and $(b-R/2)$, where R is the mirror plus position tolerance radius, works well as a first approximation. Numeric evaluation of the diffraction integral is still required to accurately design the shadow width.

BINARY APODIZATION

Up to this point we have worked with circularly symmetric apodization functions. It was assumed that the starshade could be made partially transmitting as a function of radius. While this is possible in theory, in practice a transmissive optic would ruin the ability to make the shadow very deep. Any error in transmission percentage, any variation in thickness, or any internal reflection would cause scatter that is severe on the scale we require. The solution is to use a binary mask, essentially one that is made of opaque material, patterned to simulate the needed transmission function. Petals, tapered to mimic the transmission function can provide an adequate approximation to circularly symmetric.

Because the opacity is complete out to some radius, the petals do not begin until that radius. Their widths taper in proportion to the apodization function, eventually reaching a sharp point where the exponential function has decreased the opacity to nearly zero.

We can examine the effects of the petal approximation mathematically. Knowing that circularly symmetric apodization creates the dark shadow that is needed, we need only look at the difference between that integral and the integral over the petals. This remainder must be small.

In the petal analysis we cannot perform the analysis on-axis and then extend to the surrounding areas because of circular symmetry. The integral around any circle that is centred on both the starshade and the direction to the source will be the same on axis, so even a single petal would lead to good performance at the central point. For that reason we return to equation (2), the integral as it appears at a point on the x-axis, a distance s from the centre. Subtracting away the circularly symmetric form we can write the remainder as:

$$R = \frac{k}{2pF} \int_a^\infty e^{\frac{ikr^2}{2F}} r \int_0^{2p} \left[A(\mathbf{r}, \mathbf{q}) e^{\frac{-ikrs \cos \mathbf{q}}{F}} - A(\mathbf{r}) e^{\frac{-ikrs \cos \mathbf{q}}{F}} \right] d\mathbf{q} d\mathbf{r} \quad (28)$$

where $A(\rho)$ is the circularly symmetric apodization function and $A(\rho, \theta)$ is the binary petal version of the apodization function.

Inspection of this integral shows that, as long as $\cos\theta$ does not change by much across any one petal, the value of R will be small. Proper evaluation is complicated and has been performed with a computer for some typical cases of interest. Surprisingly, a few dozen petals usually suffices. In some cases, the number of petals can be reduced as far as twelve without compromising the 10^{-10} required for planet finding.

TOLERANCING

So far, we have treated the starshade concept as a mathematical construct, without regard to whether it has any practical application. If it is ever to be built, then the tolerances for fabrication must be investigated. Any device in which the tolerances are impractically tight would be of little value. Since the starshade concept is insensitive to wavelength and to rearrangement into petals, the presumption is that the tolerances will not be particularly difficult to achieve.

In this section we address some of the more obvious kinds of errors that would be encountered in using a starshade. These errors are generic, and detailed tables of tolerances must await an actual design.

For the purposes of this tolerance analysis we shall assume that the z -axis is the direction from the star to the starshade to the detector. Thus the starshade lies in the x - y plane.

Lateral Position: For this we mean the position of the detector in the x - y direction relative to the line that extends from the source through the centre of the starshade. If the telescope drifts too far laterally it will start to leave the shadow. This distance is set by the size of the shadow. The depth of the shadow increases as one approaches the center, and the telescope must be smaller than the diameter of the region with sufficient contrast. This region becomes larger as the shade becomes larger and more distant. Thus, an optimized starshade would fit the shadow size to the telescope size. So, a margin of 20% on the starshade size appears reasonable. Thus we simply choose $\pm 0.1\lambda$ as the constraint on lateral position.

Depth of Focus: By this we mean the position of the detector on the z-axis, the line from the star through the centre of the starshade. Because of the insensitivity of the design to scaling by wavelength, it is similarly insensitive to scaling by distance. Equation (27) relates the depth of the shadow to the distance, F , through the dimensionless parameters, α and β . Since each scales as the square root of wavelength times F , the tolerance on F is set by the tolerance on λ . At the long wavelength end, the performance of the starshade degrades rapidly, so the design actually starts with the long wavelength constraint. Assuming that a ten percent degradation in wavelength is acceptable, so is a 10% change in distance. Since a typical design places the starshade at 50,000km, the depth of focus is effectively 5000km, rather easy to implement.

Rotational: Because of the circular symmetry built into the design, there is no constraint on θ_z , the rotation angle about the line of sight. Sometimes it might be better to actually spin the starshade about this axis to smooth out residual diffraction effects.

Pitch and Yaw: Because of the rotational symmetry the constraint on errors in alignment about the pitch axis, θ_x and yaw axis, θ_y , may be combined into a single pointing error. It turns out that the design is highly forgiving of such errors, but the proof takes some calculation.

We assume that the shade is out of alignment with the axis of symmetry by an angle φ about the y-axis, such that the shade appears foreshortened in the x direction by a factor of $\cos\varphi$, which we shall approximate by $1-\varepsilon$. The net optical path difference is small, about $(a+b)\theta\varphi^2/2$ for small θ and φ . As long as φ is $\ll 1$ the net path delay is a small fraction of a wavelength and may be ignored.

We start by rewriting equation (2) for the on-axis ($s=0$) case in Cartesian coordinates with the integration now taking place over the projected area which is foreshortened in one dimension

$$E = \frac{k}{2\pi i F} e^{ikF} \left[\int e^{\frac{ikx^2}{2F}} \int e^{\frac{iky^2}{2F}} dx dy + \int e^{\frac{ikx^2}{2F}} \int e^{\frac{iky^2}{2F}} e^{-\left(\frac{\sqrt{x^2+y^2}-a}{b}\right)^n} dx dy \right] \quad (29)$$

By a change of coordinate to $z=x/(1-\epsilon)$ we have

$$E = \frac{k}{2\pi i F} e^{ikF} \left[\begin{aligned} & (1-\epsilon) \int e^{\frac{iky^2}{2F}} \int e^{\frac{ikz^2(1-\epsilon)^2}{2F}} dydz + \\ & (1-\epsilon) \int e^{\frac{iky^2}{2F}} \int e^{\frac{ikz^2(1-\epsilon)^2}{2F}} e^{-\left(\frac{\sqrt{y^2+z^2(1-\epsilon)^2-a}}{b}\right)^n} dydz \end{aligned} \right] \quad (30)$$

where the integration is now over a circularly symmetric shape as before. Converting to polar coordinates we find

$$E = \frac{k}{2\pi i F} e^{ikF} \left[\begin{aligned} & (1-\epsilon) \int_0^a \int_0^{2\pi} e^{\frac{ikr^2}{2F}} e^{-\frac{ikr^2 \cos^2 q (2\epsilon - \epsilon^2)}{2F}} r dr dq + \\ & (1-\epsilon) \int_0^\infty \int_0^{2\pi} e^{\frac{ikr^2}{2F}} e^{-\frac{ikr^2 \cos^2 q (2\epsilon - \epsilon^2)}{2F}} e^{-\left(\frac{\sqrt{r^2 - r^2 \cos^2 q (2\epsilon - \epsilon^2) - a}}{b}\right)^n} r dr dq \end{aligned} \right] \quad (31)$$

Converting to polar coordinates, expanding and ignoring terms in ϵ^2 and higher, then differencing from the unperturbed integral we have an expression for the remainder caused by the misalignment:

$$\begin{aligned} R &= \frac{k}{2\pi i F} \int_0^a \int_0^{2\pi} e^{\frac{ikr^2}{2F}} \left[1 - (1-\epsilon) e^{-\frac{ik2\epsilon r^2 \cos^2 q}{2F}} \right] r dr dq \\ &+ \frac{k}{2\pi i F} \int_0^\infty \int_0^{2\pi} e^{\frac{ikr^2}{2F}} e^{-\left(\frac{r-a}{b}\right)^n} \left[1 - (1-\epsilon) e^{-\frac{ik2\epsilon r^2 \cos^2 q}{2F}} e^{-\frac{2n\epsilon r^2 \cos^2 q}{b r} \left(\frac{r-a}{b}\right)^{n-1}} \right] r dr dq \end{aligned} \quad (32)$$

Approximation of the exponentials in the brackets and dropping higher order terms reduces this to:

$$\begin{aligned} R &= \frac{k^2 \epsilon}{2F^2} \left[\int_0^a e^{\frac{ikr^2}{2F}} r^3 dr + \int_a^\infty e^{\frac{ikr^2}{2F}} e^{-\left(\frac{r-a}{b}\right)^n} r^3 dr \right] \\ &- \frac{n k \epsilon}{F b} \int_a^\infty e^{\frac{ikr^2}{2F}} e^{-\left(\frac{r-a}{b}\right)^n} r^2 \left(\frac{r-a}{b}\right)^{n-1} dr \end{aligned} \quad (33)$$

To evaluate this we use our usual change of variable:

$$\mathbf{a} = a\sqrt{\frac{k}{F}} \quad \mathbf{b} = b\sqrt{\frac{k}{F}} \quad x = \mathbf{r}\sqrt{\frac{k}{F}} \quad x = y\mathbf{b} + \mathbf{a} \quad (34)$$

and integrate by parts. The higher order terms cancel as before, leaving us with an expression for the remainder

$$R = n\mathbf{e} \int_0^{\infty} e^{\frac{i(\mathbf{b}y+\mathbf{a})^2}{2}} e^{-y^n} y^{n-1} dy \quad (35)$$

which is a form similar to that encountered in the original integral over the unperturbed aperture. We may therefore conclude that, to first order,

$$R = \mathbf{e}R_{cs} \quad (36)$$

where R_{cs} is the remaining electric field in the original circularly symmetric case. We conclude that misalignments of axis have almost no effect. They create a scale change in one axis that has no significant effect. Many degrees of misalignment can be tolerated, but in a practical mission it is likely that control issues will dictate pointing to a few arcminutes anyway.

Azimuthal Errors in Petal Shape: When we gathered the apodization function into the petals to make the function binary, we significantly perturbed the distribution of the electric field in the azimuthal direction. The total, when integrated over the circle at any given value of ρ , remained unchanged. Thus, within the azimuthal sector of width $2\pi/N$ radians at any fixed radius ρ , we are free to move the obscuration around. Essentially, the starshade is insensitive to shear in the azimuthal direction. We must simply keep the shear from slipping into the region of the adjacent petals.

Radial Errors in Petal Shape: If the petal is stretched or compressed such that the smoothness of the fall of the apodization is maintained, then there is little impact on the performance. This is reflected in the insensitivity to alignment, wherein the petals in some directions are changed in projected length, but there is no noticeable impact on

performance. Similarly the petal analysis shows that each petal independently creates its own deep shadow zone. Hence, radial scaling of modest amounts does not hurt the performance.

Truncation of Petals: Mathematically, the apodization carries out to infinity. In the case of a binary mask, this means that petals extend to infinity, something which clearly cannot be done in practice. At what radius is it safe to truncate the petal? We can write the remainder of the electric field created by truncating at a radius T.

$$R = \int_T^{\infty} e^{\frac{ix^2}{2}} e^{-\left(\frac{x-a}{b}\right)^n} x dx \quad (37)$$

which is definitely less than

$$R = \frac{1}{N} e^{-\left(\frac{T-a}{b}\right)^n} \quad (38)$$

per petal. The remainder due to truncation can be safely ignored in a typical case when the thickness of each petal has fallen below about 0.1mm. Thus the petals must be sharp at their tips, but do not have to be controlled at a microscopic level.

Area Change: Finally we consider the case where the shape changes in a discontinuous manner. Since there are many possible classes of such error, we can only address them as a generality. Consider a petal that is missing a chunk along one edge. The missing part can be contained within one half zone or spread over several. To the extent that the missing area is monotonic across the zones, the net effect is less than the largest area within one half zone. So, the size of the missing area must be less than 10^{-5} of the starshade area, but can be substantially larger if spread over several zones.

Opacity: The shade must be opaque to the needed level. If the star is to be suppressed to better than a ratio S, then the shade must transmit less than 1/S of the incident radiation

Pinholes: The presence of pinholes can simulate a level of transparency. By the Fresnel integral we see that the area of the pinholes must represent 1/S of the area of the

starshade if uniformly distributed. If contained in one zone, they must add up to less than $1/\sqrt{S}$ of the area of that zone.

Large Holes: A single large hole can be restricted to a single zone. Since a zone has an area

$$A_z = \mathbf{pl} F \tag{39}$$

the hole must have an area less than

$$A_{Hole} < \frac{\mathbf{pl} F}{\sqrt{S}} \tag{40}$$

and for typical cases the area of the hole can be as large as a square centimetre, well within a practical range.

1. Hecht, E., Optics, Addison Wesley, 4th Edition (2001).
2. Vanderbei, R.J., Spergel, D.N., and Kasdin N.J., Circularly Symmetric Apodization via Star-shaped Masks, *Astrophysical Journal*, **599**, 686-694 (2003).

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